Risks for the Long-Run and the Time-Series of Asset Returns

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Abstract

This paper combines Epstein-Zin preferences and the consumer's budget constraint to derive a relationship where the importance of the risks for the long-run can help explaining risk premium.

We find that when consumption growth, the consumption-wealth ratio and its first-differences are used as conditioning information for the Consumption Capital Asset Pricing Model (C-CAPM), the resulting linear factor model explains a large fraction of the variation in observed real stock returns for a set of sixteen OECD countries.

The model captures: (*i*) the preference of investors for a smooth path for consumption as implied by the intertemporal budget constraint; and (*ii*) the low intertemporal elasticity of substitution and the high risk aversion, which imply that agents demand large equity risk premia because they fear a reduction in future economic prospects.

Keywords: Epstein-Zin preferences, intertemporal budget constraint, expected returns, consumption capital asset pricing.

JEL classification: E21, E24, G12.

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1 Introduction

The natural economic explanation for differences in expected returns across assets is differences in risk. Lucas (1978) and Breeden (1979) argue that the risk premium on an asset is determined by its ability to insure against consumption fluctuations and Sharpe (1964) and Lintner (1965) show that the exposure of asset returns to movements in aggregate consumption explains cross-sectional differences in risk premia.

Identifying the economic sources of risks remains, however, an important economic issue because differences in the covariance of returns and contemporaneous consumption growth across portfolios have not proved to be sufficient to justify the differences in expected returns observed in the U.S. stock market (Mankiw and Shapiro, 1986; Breeden et al., 1989; Campbell, 1996; Cochrane, 1996; Lettau and Ludvigson, 2001b).Additionally, Hansen and Singleton (1982) - for the consumption-based models -, and Fama and French (1992) - for the CAPM -, show that these models have considerable difficulty in supporting the differences in a cross-section of asset returns.

The empirical failure of the canonical consumption-based asset pricing model has spawned a large literature that addresses its shortcomings: inefficiencies of financial markets (Fama (1970, 1991, 1998), Fama and French (1996), Farmer and Lo (1999)); the rational response of agents to time-varying investment opportunities driven by variation in risk aversion (Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999)) or in the joint distribution of consumption and asset returns (Duffee (2005), Santos and Veronesi (2006)) have been offered as explanations for why differences in expected returns are not due to differences in risk to consumption. In addition, several papers tried to shed more light on this question and many economically motivated variables have been developed to capture time-variation in expected returns and document long-term predictability.[1]

[1] See, for example, Fama and French (1988), Campbell and Shiller (1988), Poterba and Summers (1988), Richards (1995), Lettau and Ludvigson (2001a, 2004). Lettau and Ludvigson (2001a) show that the transitory deviation from the common trend in consumption, aggregate wealth and labor income, *cay*, is a strong predictor of asset returns, as long as the expected return to human capital and consumption growth are not too volatile. Fernandez-Corugedo et al. (2003) use the same approach but incorporate the relative price of durable goods, whilst Julliard (2004) shows that the expected changes in labor income are important because of their ability to track time varying risk premia. The nonseparability between consumption and leisure in on the basis of the work of Wei (2005), who argue that human capital risk can generate sufficient variation in the agent's risk attitude to produce equity returns and bond yields with properties close to the observed in the data. Whilst the last two papers emphasize the role of human capital, others have focused on the importance of the housing market instead. Yogo (2006) and Piazzesi et al. (2007) emphasize the role of nonseparability of preferences in explaining the countercyclical variation in the equity premium. Pakos (2003) argues that there is an important non-homotheticity in preferences. In the same spirit, Lustig and Van Nieuwerburgh (2005) show that the ratio of housing wealth to human wealth (the housing collateral ratio) shifts the conditional distribution of asset prices and

Within the representative agent representation, two main lines of investigation have been successfully explored. The first approach introduces time-varying risk-aversion in preferences and is based on the external habit model of Campbell and Cochrane (1999), which was designed to show that equilibrium asset prices can match the data in a world without predictability in cash-flows, that is, where dividend growth and aggregate consumption are *i.i.d.*.[2] The second approach is based on the concept of long-run risk (Epstein and Zin, 1991; Bansal and Yaron, 2004), and introduces predictability in aggregate consumption growth, as a result of the persistency of the shocks to cash-flows.[3] Low-frequency movements, and time-varying uncertainty in aggregate consumption growth are the key channels for understanding asset prices.

The model of the long-run risk of Bansal and Yaron (2004) has two major features. First, it relies in the Epstein and Zin (1989) preferences, which allows for a separation between the intertemporal elasticity of substitution and risk aversion. Second, it models consumption and dividend growth are modelled as containing a small persistent expected growth component, and fluctuating volatility, which captures time-varying economic uncertainty. The authors show that an intertemporal elasticity of substitution greater than 1 is critical for capturing the observed negative correlation between consumption volatility and price-dividend ratios. The results show that risks related to varying growth prospects and fluctuating economic uncertainty can quantitatively justify many of the observed features of asset market data.

This paper combines Epstein-Zin preferences, the intertemporal budget constraint and the homogeneity property of the Bellman equation to derive a relationship that highlights the role of risks for the long-run in predicting U.S. quarterly stock market returns.

We explore this relationship to check whether it carries relevant information to predict future asset

consumption growth and, therefore, predicts returns on stocks.

[2] Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Abel (1999) are among the early contributions to the literature on habit-formation models. On the other hand, Menzly et al. (2004) and Wachter (2006) provide more recent approaches to the topic. Chen and Ludvigson (2007) estimate the habit process for a class of external habit models. Sousa (2007) tests the CRRA assumption using macroeconomic data, and shows that the representative agent may have habit-formation preferences.

[3] Bansal et al. (2005) suggest that changes in expectations about the entire path of future cash flows can account for the puzzling differences in risk premia across book-to-market, momentum, and size-sorted portfolios. Hansen et al. (2006), Parker and Julliard (2005) and Malloy et al. (2005) measure long-run risk based on leads and long-run impulse responses of consumption growth. Bansal et al. (2006) estimate the long-run risk model, whilst Piazzesi and Schneider (2006) study its implications for the yield curve, Bansal et al. (2005) and Yang (2007) study the implications for the cross-section of equity portfolios, and Benzoni et al. (2005) for credit spreads. Bekaert et al. (2005) estimate both long-run risk and external habit models. Bansal et al. (2007) estimate and examine the empirical plausibility of the habit-formation model and the long-run risk model.

returns and show that the implied stochastic discount factor can be expressed as a function of the consumption growth, C_{t+1}/C_t, the consumption-aggregate wealth ratio, *cay*, and its first-differences, Δcay.

We find that: (i) risks for the long-run are a important determinants of both real returns and asset returns over a Treasury bill rate; and (ii) when risks for the long-run are used as conditioning information for the Consumption Capital Asset Pricing Model (C-CAPM), the resulting linear factor model explains a large fraction of the variation in real stock returns for a panel of sixteen OECD countries. In particular, the predictive ability of the model with regard to future real stock returns is stronger for Australia, Belgium and US (both 9%), Canada (13%), Denmark (17%), Finland (15%), France (21%) and UK (23%) at the 4-quarter ahead horizon. The results are robust to the inclusion of additional control variables and outperform the *cay* (Lettau and Ludvigson, 2001) and the *cday* (Sousa, 2010) models.

The success of the model in predicting asset returns is due to its ability to track time varying equilibrium risk premia. The model captures: (i) the preference of investors for a smooth path for consumption as implied by the intertemporal budget constraint; and (ii) the separation between a low intertemporal elasticity of substitution and a high risk aversion, implying that agents demand large equity risk premia because they fear that a reduction in economic prospects or a rise in economic uncertainty will lower asset prices. The risks for the long-run are, therefore, important determinants of the risk premium and explain a substantial fraction of the time-series variation that one observes in expected returns.

The paper is organized as follows. Section 2 presents the theoretical approach and how we combine the Epstein-Zin preferences with the intertemporal budget constraint to derive the stochastic discount factor. Section 3 describes the data and discusses the empirical results. Section 4 concludes and discusses the implications of the findings.

2 Epstein-Zin Preferences and the Intertemporal Budget Constraint

Consider a representative agent economy in which wealth is tradable. Defining W_t as time t aggregate wealth (human capital plus asset wealth), C_t as time t consumption and $R_{w,t+1}$ as the return on aggregate wealth between period t and $t+1$, the consumer's budget constraint can be written as[4]

[4] Labor income does not appear explicitly in this equation because of the assumption that the market value of tradable human capital is included in aggregate wealth.

$$W_{t+1} = (1 + R_{t+1})(W_t - C_t) \quad \forall t \tag{1}$$

where W_t is total wealth and $R_{w,t}$ is the return on wealth, that is,

$$R_{t+1} := \left(1 - \sum_{i=1}^{N} w_{it}\right) R^f + \sum_{i=1}^{N} w_{it} R_{it+1} = R^f + \sum_{i=1}^{N} w_{it} \left(R_{it+1} - R^f\right) \tag{2}$$

where w_i is the wealth share invested in the ith risky asset and R^f is the risk-free rate.

With Epstein and Zin (1989, 1991) preferences, the optimal value of the utility, V, at time t will be a function of the wealth W_t and takes the form

$$V(W_t) \equiv \max_{\{C,w\}} \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{3}$$

where C_t is the consumption, δ is the rate of time preference, γ is the relative risk aversion, ψ is the intertemporal elasticity of substitution, E_t is the rational expectation operator, and $\theta := \frac{1-\gamma}{1-1/\psi}$.

By homogeneity, $V(W_t) \equiv \phi_t W_t$ for some ϕ_t and, given the structure of the problem, consumption is also proportional to W_t, that is $C_t = \varphi_t W_t$.

The first-order condition for C_t can be written as

$$\delta E_t \left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} = (1-\delta) \left(\frac{\varphi_t}{1-\varphi_t} \right)^{\frac{1-\gamma}{\theta}-1}. \tag{4}$$

Using homogeneity, equation(3) becomes:

$$\begin{aligned} \phi_t &= \max \left\{ (1-\delta) \left(\frac{C_t}{W_t} \right)^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \left(1 - \frac{C_t}{W_t} \right)^{\frac{1-\gamma}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ &= (1-\delta)^{\frac{\theta}{1-\gamma}} \left(\frac{C_t}{W_t} \right)^{1-\frac{\theta}{1-\gamma}}. \end{aligned}$$

Plugging the solution for ϕ_t in the first-order condition (4), one can derive the Euler equation for the return on wealth

$$1 = E_t \left[\delta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{t+1}^\theta \right] \quad \forall t. \tag{5}$$

The first-order condition for w_{it} can be written as

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{t+1}^{\theta-1} R_{it+1} \right] = E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{t+1}^{\theta-1} \right] R^f \quad \forall t, i. \tag{6}$$

From the Euler equation (5) and the definition of return on wealth (2), we have

$$1 = E_t \left[\delta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{t+1}^{\theta-1} \left(R^f + \sum_{i=1}^{N} w_{it} \left(R_{it+1} - R^f \right) \right) \right] \quad \forall t.$$

Using (6), the equilibrium risk free rate is such that:

$$1/R^f = E_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{t+1}^{\theta-1}\right] \quad \forall t.$$

Finally, multiplying both sides of (6) by δ^{θ} and using the last result to remove R^f, the Euler equation for any risky asset i becomes:

$$E_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{t+1}^{\theta-1} R_{it+1}\right] = 1 \quad \forall t, i. \tag{7}$$

From equation (1), one obtains

$$R_{t+1}^{-1} = \frac{W_t}{W_{t+1}} - \frac{C_t}{W_{t+1}} = \frac{C_t}{C_{t+1}}\left(\frac{W_t}{C_t}\frac{C_{t+1}}{W_{t+1}} - \frac{C_{t+1}}{W_{t+1}}\right)$$

and consequently,

$$R_{t+1}^{\theta-1} = e^{(\theta-1)\Delta c_{t+1}}\left[e^{\Delta cw_{t+1}} - e^{cw_{t+1}}\right]^{1-\theta}$$

where $cw_t := \log\left(C_t/W_t\right)$.

Putting the last result into equation (7), we have

$$E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[e^{\Delta cw_{t+1}} - e^{cw_{t+1}}\right]^{1-\theta}\left(R_{it+1} - R^f\right)\right\} = 0$$

where the stochastic discount factor, m_t is:[5]

$$m_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[e^{\Delta cw_{t+1}} - e^{cw_{t+1}}\right]^{1-\frac{1-\gamma}{1-1/\psi}} \tag{8}$$

In order to estimate the last equation, we need a proxy for cw. Following Lettau and Ludvigson (2001a)

$$cw_t \approx \kappa + cay_t.$$

Consequently, the empirical moment function can be expressed as

$$E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[e^{\Delta cay_{t+1}} - e^{\kappa+cay_{t+1}}\right]^{1-\frac{1-\gamma}{1-1/\psi}}\left(R_{it+1} - R_{t+1}^f\right)\right\} = 0$$

or

$$E\left[g\left(\mathrm{R}_t^e, \frac{C_{t+1}}{C_t}, \Delta cay_{t+1}, cay_{t+1}; \mu, \gamma, \alpha, \kappa, \psi\right)\right] = 0. \tag{9}$$

Similarly, equation (8) can be written as:

$$m_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[e^{\Delta cay_{t+1}} - e^{\kappa+cay_{t+1}}\right]^{1-\frac{1-\gamma}{1-1/\psi}}. \tag{10}$$

[5] Appendices B and C provide the derivation of the stochastic discount factor.

In this paper, we look at the time-series properties of asset returns. More specifically, the Epstein-Zin preferences are combined with the intertemporal budget constraint and, using the homogeneity of the Bellman equation, we show that one can derive a relationship between excess returns, consumption growth, cay and Δcay. The model says that the expected returns is the weighted sum of the covariance of the return and each factor. Denoting the vector of factors by f_{t+1}, and combining Epstein-Zin preferences with cay to recover the return on wealth, we get:

$$f_{t+1} = (\frac{C_{t+1}}{C_t}, \Delta cay_{t+1}, cay_{t+1})'. \tag{11}$$

As a result, both the consumption-wealth ratio, cay, and its first-differences, Δcay, can be used to predict future asset returns.

3 Do Risks for the Long Run Explain Asset Returns?

3.1 Data

In the estimation of the long-run relationships among consumption, (dis)aggregate wealth and labour income, we use quarterly data, post-1960, for 16 countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Spain, Sweden, the UK, the US).

The consumption data are the private consumption expenditure and were taken from the database of the NiGEM model of the NIESR Institute, the Main Economic Indicators of the OECD and DRI International. The labour income data correspond to the compensation series of the NIESR Institute. In the case of the US, labour income series was constructed following Lettau and Ludvigson (2004). The wealth data were taken from the national central banks or Eurostat. For the G-8 countries, the wealth series were compared with alternative sources, namely, Bertaut (2002), Pichette and Tremblay (2003), Tan and Voss (2003), Catte et al. (2004), and the Bank of Japan.

The stock return data were computed using the share price index and the dividend yield ratio provided by the International Financial Statistics of the IMF and the Datastream.

Finally, the population series were taken from the OECD's Main Economic Indicators and interpolated (from annual data), and all series were deflated with consumption deflators and expressed in logs of per capita terms (with the obvious exception of the excess returns). The series were seasonally adjusted using the X-12 method where necessary and the time frames were chosen based on the availability of reliable data for each country.

3.2 The Long-Run Relationship Between Consumption, Asset Wealth and Labour Income

As a preliminary step, we test for unit roots in consumption, aggregate wealth and labour income using the Augmented Dickey-Fuller and the Phillips-Perron tests. These show that the three variables are integrated of order one. Then, we apply the Engle-Granger test for cointegration. Finally, following Stock and Watson (1993) we estimate the equation below with dynamic least squares (DOLS):

$$c_t = \mu + \beta_a a_t + \beta_y y_t + \sum_{i=-k}^{k} b_{a,i} \Delta a_{t-i} + \sum_{i=-k}^{k} b_{y,i} \Delta y_{t-i} + \varepsilon_t, \tag{12}$$

where the parameters β_a and β_y represent, respectively, the long-run elasticities of consumption with respect to asset wealth and labor income, Δ denotes the first difference operator, μ is a constant, and ε_t is the error term.

Since the impact of different assets' categories on consumption can vary (Poterba and Samwick, 1995; Sousa, 2010), we also disaggregate wealth into its main components: financial wealth and housing wealth. We use specify the following equation

$$c_t = \mu + \beta_f f_t + \beta_u u_t + \beta_y y_t + \sum_{i=-k}^{k} b_{f,i} \Delta f_{t-i} + \sum_{i=-k}^{k} b_{u,i} \Delta u_{t-i} + \sum_{i=-k}^{k} b_{y,i} \Delta y_{t-i} + \varepsilon_t, \tag{13}$$

where the parameters β_f, β_u,β_y represent, respectively, the long-run elasticities of consumption with respect to financial wealth, housing wealth, and labor income, Δ denotes the first difference operator, μ is a constant, and ε_t is the error term.

Table 1 shows the estimates for the shared trend among consumption, asset wealth, and income, cay_t. It can be seen that, despite some heterogeneity, the long-run elasticities of consumption with respect to aggregate wealth and labour income imply roughly shares of one third and two thirds for asset wealth and human wealth, respectively. This is particularly true for Australia, Canada, Finland, France, Ireland, the UK and the US. Moreover, the disaggregation between asset wealth and labour income is statistically significant for all countries (with the exceptions of Finland and Italy).

Table 1: The long-run relationship between consumption, asset wealth and labour income.

Australia	$cay_t := c_t - 0.35^{***}a_t - 0.54^{***}y_t$	Italy	$cay_t := c_t + 0.02a_t - 1.49^{***}y_t$
Belgium	$cay_t := c_t - 0.16^{***}a_t - 0.56^{***}y_t$	Japan	$cay_t := c_t - 0.08^{***}a_t - 0.89^{***}y_t$
Canada	$cay_t := c_t - 0.36^{***}a_t - 0.56^{***}y_t$	Netherlands	$cay_t := c_t - 0.17^{***}a_t - 0.53^{***}y_t$
Denmark	$cay_t := c_t - 0.09^{***}a_t - 0.65^{***}y_t$	Spain	$cay_t := c_t - 0.06^{*}a_t - 0.76^{***}y_t$
Finland	$cay_t := c_t - 0.38^{***}a_t - 0.13y_t$	Sweden	$cay_t := c_t + 0.13^{**}a_t - 1.12^{***}y_t$
France	$cay_t := c_t - 0.25^{***}a_t - 0.55^{***}y_t$	UK	$cay_t := c_t - 0.32^{***}a_t - 0.66^{***}y_t$
Germany	$cay_t := c_t - 0.13^{*}a_t - 1.16^{***}y_t$	US	$cay_t := c_t - 0.28^{***}a_t - 0.79^{***}y_t$
Ireland	$cay_t := c_t - 0.36^{***}a_t - 0.46^{***}y_t$		

*, **, *** denote statistical significance at the 10, 5, and 1% level, respectively.

Table 2: The long-run relationship between consumption, financial wealth, housing wealth and labour income.

Australia	$cday_t := c_t - 0.07^{***}f_t - 0.27^{***}u_t - 0.59^{***}y_t$
Austria	$cday_t := c_t + 0.05^{***}f_t - 0.02u_t - 1.44^{***}y_t$
Belgium	$cday_t := c_t - 0.11^{***}f_t + 0.06^{**}u_t - 0.80^{***}y_t$
Canada	$cday_t := c_t - 0.30^{***}f_t - 0.06^{***}u_t - 0.49^{***}y_t$
Denmark	$cday_t := c_t - 0.02^{**}f_t - 0.02u_t - 0.71^{***}y_t$
Finland	$cday_t := c_t - 0.14^{***}f_t + 0.04u_t + 0.69^{***}y_t$
France	$cday_t := c_t - 0.08^{***}f_t - 0.10^{***}u_t - 0.62^{***}y_t$
Germany	$cday_t := c_t - 0.31^{***}f_t - 0.09^{***}u_t - 0.33^{***}y_t$
Ireland	$cday_t := c_t + 0.13^{***}f_t - 0.13^{***}u_t - 0.53^{***}y_t$
Italy	$cday_t := c_t - 0.24^{***}f_t + 0.03u_t - 0.74^{***}y_t$
Japan	$cday_t := c_t - 0.17^{***}f_t + 0.03u_t - 0.86^{***}y_t$
Netherlands	$cday_t := c_t - 0.08^{***}f_t - 0.10^{***}u_t - 0.53^{***}y_t$
Spain	$cday_t := c_t - 0.08^{***}f_t + 0.02u_t - 0.67^{***}y_t$
Sweden	$cday_t := c_t - 0.12^{***}f_t + 0.15^{***}u_t - 0.61^{***}y_t$
UK	$cday_t := c_t - 0.17^{***}f_t - 0.06^{***}u_t - 0.76^{***}y_t$
US	$cday_t := c_t - 0.04^{***}f_t + 0.02u_t - 1.21^{***}y_t$

*, **, *** denote statistical significance at the 10, 5, and 1% level, respectively.

Table 2 reports the estimates of the long-run elasticities of consumption with respect to financial wealth, housing wealth and labour income. First, it shows that the disaggregation between financial and housing wealth is statistically significant for almost all countries. Moreover, consumption is, in general, more sensitive to financial wealth than to housing wealth, as the elasticities of consumption with respect to financial wealth are larger in magnitude. Second, it tells us that consumption is very responsive to financial wealth in the case of Belgium (0.11), Canada (0.30), Finland (0.14), Germany (0.31), Italy (0.24), Sweden (0.12) and the UK (0.17). Third, the long-run elasticity of consumption with respect to housing wealth is particularly strong for Australia (0.27), France (0.10), Ireland (0.13) and the Netherlands (0.10).

3.3 Forecasting Regressions for Real Stock Returns

The model shows that both the transitory deviations from the long-run relationship among consumption, aggregate wealth and income, cay_t, and its first-differences, Δcay_t, are important conditioning variables that provide information about agents' expectations of future changes in asset returns.

Moreover, given the disaggregation of asset wealth into its main components (financial and housing wealth), we argue that $cday_t$ should provide a better forecast for asset returns than a variable like cay_t in Lettau and Ludvigson (2001). We look at real stock returns (denoted by r_t) for which quarterly data are available and should provide a good proxy for the non-human component of asset wealth.

Tables 3 and 4 summarize the forecasting power of cay and Δcay at different horizons. They reports estimates from OLS regressions of the H-period real stock return, $r_{t+1} + \ldots + r_{t+H}$, on the lag of cay_t and the lag of its first-difference, Δcay. They show that cay_t is statistically significant for almost all countries and the point estimate of the coefficient is large in magnitude. Moreover, its sign is positive. These results suggest that investors will temporarily allow consumption to rise above its equilibrium level in order to smooth it and insulate it from an increase in real stock returns. Therefore, deviations in the long-term trend among c_t, a_t and y_t should be positively related to future stock returns. As for Δcay, the evidence is somewhat weaker, as it is statistically significant for a few countries. However, it can be seen that the two variables explain an important fraction of the variation in future real returns (as described by the adjusted R-square), in particular, at horizons spanning from three to four quarters. In fact, at the four quarter horizon, cay_t explains 23% (UK), 21% (France), 17% (Denmark), 15% (Finland), 13% (Canada) and 9% (Australia, Belgium and US) of the real stock return. In contrast, its forecasting power is poor for countries such as Germany, Ireland, Spain and Sweden.

Table 3

Forecasting regressions for real stock returns (using cay).

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
Regressor	1	2	3	4	Regressor	1	2	3	4
Australia					Denmark				
cay_t	0.54	0.96*	1.39**	1.77**	cay_t	0.44***	0.91***	1.40***	1.91***
(t-stat)	(1.47)	(1.82)	(2.24)	(2.48)	(t-stat)	(3.08)	(3.75)	(4.19)	(4.58)
Δcay_t	1.65**	2.11*	1.58	2.61	Δcay_t	-0.42	-0.70	-0.72	-1.14*
(t-stat)	(2.15)	(1.66)	(1.03	(1.42)	(t-stat)	(-1.42)	(-1.60)	(-1.37)	(-1.73)
$\bar{R}^2$	[0.07]	[0.14]	[0.07]	[0.09]	$\bar{R}^2$	[0.09]	[0.12]	[0.14]	[0.17]
		Austria					Finland		
cay_t	0.56	1.06	1.61*	2.07*	cay_t	0.87**	1.85***	2.85***	3.82***
(t-stat)	(1.35)	(1.49)	(1.67)	(1.69)	(t-stat)	(2.39)	(3.37)	(4.01)	(4.30)
Δcay_t	-0.26	-0.44	-0.98	-0.57	Δcay_t	-2.29	-3.57*	-3.77	-3.85
(t-stat)	(-0.36)	(-0.39)	(-0.68)	(-0.28)	(t-stat)	(-1.58)	(-1.79)	(-1.44)	(-1.12)
$\bar{R}^2$	[0.02]	[0.03]	[0.03]	[0.04]	$\bar{R}^2$	[0.07]	[0.11]	[0.13]	[0.15]
		Belgium					France		
cay_t	1.68***	2.84***	3.20***	2.71**	cay_t	1.71***	3.41***	4.87***	6.30***
(t-stat)	(4.02)	(3.83)	(2.87)	(2.12)	(t-stat)	(3.15)	(4.09)	(4.79)	(5.44)
Δcay_t	0.46	1.32	2.77	4.38**	Δcay_t	-1.96**	-1.84	-1.85	-2.65
(t-stat)	(0.52)	(0.89)	(1.51)	(2.05)	(t-stat)	(-2.12)	(-1.39)	(-1.08)	(-1.32)
$\bar{R}^2$	[0.10]	[0.13]	[0.12]	[0.09]	$\bar{R}^2$	[0.10]	[0.14]	[0.18]	[0.21]
		Canada					Germany		
cay_t	0.66***	1.08***	1.20**	1.18*	cay_t	-0.27	-0.52	-0.77	-1.12*
(t-stat)	(2.89)	(2.80)	(2.28)	(1.82)	(t-stat)	(-1.01)	(-1.22)	(-1.42)	(-1.72)
Δcay_t	0.63	2.09**	3.39**	3.48**	Δcay_t	0.38	0.73	1.43	1.60
(t-stat)	(1.03)	(1.95)	(2.41)	(2.25)	(t-stat)	(0.94)	(0.98)	(1.50)	(1.37)
$\bar{R}^2$	[0.07]	[0.12]	[0.13]	[0.10]	$\bar{R}^2$	[0.01]	[0.02]	[0.03]	[0.03]

Table 4

Forecasting regressions for real stock returns (using cay) (cont.).

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
Regressor	1	2	3	4	Regressor	1	2	3	4
	Ireland					Spain			
cay_t	0.13	-0.18	-0.64	-0.48	cay_t	-0.38	0.04	0.10	-0.02
(t-stat)	(0.20)	(-0.16)	(-0.45)	(-0.30)	(t-stat)	(-0.40)	(0.03)	(0.08)	(-0.01)
Δcay_t	0.92	1.63	0.91	-0.81	Δcay_t	-0.31	-0.42	-0.81	0.84
(t-stat)	(0.98)	(0.92)	(0.40)	(-0.30)	(t-stat)	(-0.13)	(-0.13)	(-0.21)	(0.20)
$\bar{R}^2$	[0.01]	[0.01]	[0.00]	[0.00]	$\bar{R}^2$	[0.00]	[0.00]	[0.00]	[0.00]
	Italy					Sweden			
cay_t	0.24	0.43	0.60	0.86	cay_t	0.05	0.03	-0.22	-0.59
(t-stat)	(0.96)	(1.13)	(1.27)	(1.56)	(t-stat)	(0.19)	(0.07)	(-0.37)	(-0.83)
Δcay_t	-0.17	0.10	1.55	1.12	Δcay_t	-0.03	1.01	3.31	3.29
(t-stat)	(-0.15)	(0.05)	(0.60)	(0.36)	(t-stat)	(-0.03)	(0.65)	(1.38)	(1.22)
$\bar{R}^2$	[0.01]	[0.01]	[0.02]	[0.03]	$\bar{R}^2$	[0.00]	[0.01]	[0.054]	[0.03]
Japan					UK				
cay_t	0.77*	1.19*	1.35	1.46	cay_t	1.00***	1.89***	2.63***	3.07***
(t-stat)	(1.83)	(1.68)	(1.44)	(1.28)	(t-stat)	(3.71)	(3.90)	(4.09)	(4.31)
Δcay_t	0.16	0.82	0.52	1.33	Δcay_t	-0.47	-0.14	0.35	2.09
(t-stat)	(0.29)	(0.84)	(0.43)	(0.82)	(t-stat)	(-0.69)	(-0.14)	(0.28)	(1.56)
$\bar{R}^2$	[0.04]	[0.05]	[0.03]	[0.03]	$\bar{R}^2$	[0.10]	[0.15]	[0.19]	[0.23]
	Netherlands					US			
cay_t	0.66*	1.40**	2.28***	2.87***	cay_t	0.87	1.69***	2.36***	3.08***
(t-stat)	(1.73)	(2.46)	(3.10)	(3.26)	(t-stat)	(2.66)	(2.96)	(3.10)	(3.37)
Δcay_t	-0.01	-0.86	-1.28	-0.86	Δcay_t	-0.83	-0.77	-0.77	-1.39
(t-stat)	(-0.02)	(-0.94)	(-1.14)	(-0.59)	(t-stat)	(-1.12)	(-0.70)	(-0.57)	(-0.93)
$\bar{R}^2$	[0.02]	[0.03]	[0.05]	[0.06]	$\bar{R}^2$	[0.04]	[0.06]	[0.07]	[0.09]

Tables 5 and 6 summarize the forecasting power of $cday_t$ and its first-difference, $\Delta cday_t$, at different horizons. It reports estimates from OLS regressions of the H-period real stock return, $r_{t+1} + \ldots + r_{t+H}$, on the lag of $cday_t$ and its first-difference, $\Delta cday_t$.

In accordance with the findings for cay_t, it shows that $cday_t$ is statistically significant for almost all countries, the point estimate of the coefficient is large in magnitude and its sign is positive. Therefore, deviations in the long-term trend among c_t, f_t, u_t and y_t should be positively linked with future stock returns.

In addition, it can be seen that the trend deviations explain a substantial fraction of the variation in future real returns. At the four quarter horizon, $cday_t$ and $\Delta cday_t$ explain 24% (Belgium and France), 23% (UK), 19% (Canada), 14% (Denmark), 9% (US), 7% (Australia and Netherlands), 4% (Finland) of the real stock return. However, it does not seem to exhibit forecasting power for countries such as Germany, Ireland, and Spain.

Noticeably, it is important to emphasize that, in general, $cday_t$ performs better than cay_t, also in accordance with the findings of Sousa (2010), reflecting the ability of $cday_t$ to track the changes in the composition of asset wealth. Portfolios with different compositions of assets are subject to different degrees of liquidity, taxation, or transaction costs. For example, agents who hold portfolios where the exposure to housing wealth is larger face an additional risk associated with the (il)liquidity of these assets and the transaction costs involved in trading them. Wealth composition is, therefore, an important source of risk that $cday_t$ – but not cay_t – is able to capture.

Table 5

Forecasting regressions for real stock returns (using *cday*).

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
Regressor	1	2	3	4	Regressor	1	2	3	4
	Australia					Denmark			
$cday_t$	0.56	1.04*	1.63**	2.18**	$cday_t$	0.40***	0.83***	1.27***	1.73***
(t-stat)	(1.38)	(1.77)	(2.19)	(2.42)	(t-stat)	(2.81)	(3.39)	(3.81)	(4.16)
$\Delta cday_t$	0.90	0.61	-0.52	0.44	$\Delta cday_t$	-0.36	-0.58	-0.55	-0.91
(t-stat)	(1.31)	(0.50)	(-0.35)	(0.25)	(t-stat)	(-1.26)	(-1.39)	(-1.07)	(-1.42)
$\bar{R}^2$	[0.04]	[0.04]	[0.05]	[0.07]	$\bar{R}^2$	[0.08]	[0.11]	[0.13]	[0.14]
	Austria					Finland			
$cday_t$	0.41	0.76	1.14	1.47	$cday_t$	0.87*	1.57**	2.12**	2.61**
(t-stat)	(1.06)	(1.12)	(1.25)	(1.25)	(t-stat)	(1.73)	(1.93)	(2.06)	(2.00)
$\Delta cday_t$	-0.11	-0.18	-0.66	-0.31	$\Delta cday_t$	0.50	1.31	1.89	1.78
(t-stat)	(-0.17)	(-0.16)	(-0.47)	(-0.15)	(t-stat)	(0.34)	(0.64)	(0.68)	(0.50)
$\bar{R}^2$	[0.01]	[0.01]	[0.02]	[0.02]	$\bar{R}^2$	[0.03]	[0.04]	[0.04]	[0.04]
	Belgium					France			
$cday_t$	2.38***	4.37***	5.75***	6.40***	$cday_t$	2.15***	4.35***	6.33***	8.34***
(t-stat)	(4.45)	(5.88)	(7.26)	(7.45)	(t-stat)	(3.37)	(4.47)	(5.41)	(6.47)
$\Delta cday_t$	-0.63	-0.98	-0.59	0.23	$\Delta cday_t$	-2.25**	-2.61**	-3.29**	-4.61**
(t-stat)	(-0.87)	(-0.99)	(-0.48)	(0.14)	(t-stat)	(-2.34)	(-1.97)	(-2.04)	(-2.47)
$\bar{R}^2$	[0.18]	[0.24]	[0.27]	[0.24]	$\bar{R}^2$	[0.10]	[0.14]	[0.20]	[0.24]
	Canada					Germany			
$cday_t$	1.26***	2.29***	2.87***	3.17***	$cday_t$	-1.41**	-1.84*	-1.78	-1.74
(t-stat)	(4.54)	(4.64)	(4.05)	(3.43)	(t-stat)	(-2.18)	(-1.82)	(-1.33)	(-1.04)
$\Delta cday_t$	-0.25	0.55	1.80	1.87	$\Delta cday_t$	-0.22	-0.71	-0.64	-0.73
(t-stat)	(-0.42)	(0.54)	(1.36)	(1.23)	(t-stat)	(-0.30)	(-0.62)	(-0.43)	(-0.43)
$\bar{R}^2$	[0.12]	[0.20]	[0.22]	[0.19]	$\bar{R}^2$	[0.06]	[0.05]	[0.03]	[0.02]

Table 6

Forecasting regressions for real stock returns (using *cday*) (cont.).

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
Regressor	1	2	3	4	Regressor	1	2	3	4
	Ireland					Spain			
$cday_t$	0.21	-0.06	-0.56	-0.26	$cday_t$	-0.44	1.22	3.44	5.03*
(t-stat)	(0.32)	(-0.05)	(-0.42)	(-0.17)	(t-stat)	(-0.33)	(0.70)	(1.46)	(1.68)
$\Delta cday_t$	0.74	1.38	0.47	-1.46	$\Delta cday_t$	-1.22	-2.99	-4.85*	-3.79
(t-stat)	(0.83)	(0.80)	(0.20)	(-0.50)	(t-stat)	(-0.76)	(-1.20)	(-1.65)	(-1.22)
$\bar{R}^2$	[0.01]	[0.01]	[0.00]	[0.00]	$\bar{R}^2$	[0.02]	[0.02]	[0.05]	[0.04]
	Italy					Sweden			
$cday_t$	0.70	1.20	1.62	2.32*	$cday_t$	1.17**	2.53***	3.49***	3.93***
(t-stat)	(1.21)	(1.39)	(1.58)	(1.93)	(t-stat)	(2.10)	(3.04)	(3.68)	(3.75)
$\Delta cday_t$	-0.67	0.80	1.57	-1.27	$\Delta cday_t$	-2.16***	-2.36	-0.24	0.38
(t-stat)	(-0.36)	(0.24)	(0.34)	(-0.23)	(t-stat)	(-2.65)	(-1.56)	(-0.11)	(0.15)
$\bar{R}^2$	[0.02]	[0.02]	[0.03]	[0.03]	$\bar{R}^2$	[0.08]	[0.10]	[0.11]	[0.11]
	Japan					UK			
$cday_t$	0.72*	1.18	1.39	1.58	$cday_t$	1.00***	1.89***	2.63***	3.07***
(t-stat)	(1.70)	(1.61)	(1.43)	(1.36)	(t-stat)	(3.71)	(3.90)	(4.09)	(4.31)
$\Delta cday_t$	0.19	0.81	0.63	1.38	$\Delta cday_t$	-0.47	-0.14	0.35	2.09
(t-stat)	(0.32)	(0.80)	(0.53)	(0.85)	(t-stat)	(-0.69)	(-0.14)	(0.28)	(1.56)
$\bar{R}^2$	[0.04]	[0.04]	[0.03]	[0.04]	$\bar{R}^2$	[0.10]	[0.15]	[0.19]	[0.23]
	Netherlands					US			
$cday_t$	0.73**	1.60***	2.56***	3.22***	$cday_t$	0.87	1.69***	2.36***	3.08***
(t-stat)	(1.96)	(2.84)	(3.48)	(3.61)	(t-stat)	(2.66)	(2.96)	(3.10)	(3.37)
$\Delta cday_t$	-0.15	-1.08	-1.49	-1.11	$\Delta cday_t$	-0.83	-0.77	-0.77	-1.39
(t-stat)	(-0.28)	(-1.24)	(-1.38)	(-0.78)	(t-stat)	(-1.12)	(-0.70)	(-0.57)	(-0.93)
$\bar{R}^2$	[0.02]	[0.04]	[0.06]	[0.07]	$\bar{R}^2$	[0.04]	[0.06]	[0.07]	[0.09]

3.4 Additional control variables

Campbell and Shiller (1988), Fama and French (1988) and Lamont (1998) show that the ratios of price to dividends or earnings or the ratio of dividends to earnings have predictive power for stock returns. More recently, Goyal and Welsh (2003) argue that because the dividend yield follows a random walk it cannot predict stock prices. However, Robertson and Wright (2006) and Boudoukh et al. (2007) mention that a change in tax legislation in the US in 1983 that legalised share buybacks implies an adjustment of the dividend yield for these and similar effects. Consequently, this adjusted statistic is mean reverting and a good predictor of stock returns.

Tables 7 and 8 report the adjusted R-square statistics for two models: (i) in Panel A, the model includes cay_t only; and (ii) in Panel B, the model includes, in addition to cay_t and Δcay_t, the lagged stock returns, r_{t-1}, and the lag of the dividend yield ratio, dy.

It can be seen, that the model that includes cay_t only underperforms our model (which adds Δcay as a regressor). In fact, at the four quarter horizon, cay_t explains 20% (France), 18% (UK), 17% (Canada), 15% (Denmark), 14% (Finland), 8% (Belgium and US) and 7% (Australia) of the real stock return, which is lower than our previous findings.

When we consider additional control variables, the results show that both the point coefficient estimates of cay_t and Δcay_t and their statistical significance do not change with respect to the findings of Tables 3 and 4 where only cay and Δcay were included as explanatory variables. Moreover, the lag of the dependent variable is not statistically significant, a feature that is in accordance with the forward-looking behaviour of stock returns. Finally, the dividend yield ratio, dy, seems to provide relevant information about future asset returns since it is statistically significant in practically all regressions and it improves the adjusted R-square.

A similar conclusion can be drawn from Tables 9 and 10, where we present the predictive ability - as measured by their adjusted R-square statistics - of two models: (i) in Panel A, the model includes $cday_t$ only; and (ii) in Panel B, the model includes, in addition to $cday_t$ and $\Delta cday_t$, the lagged stock returns, r_{t-1}, and the lag of the dividend yield ratio, dy. The empirical findings corroborate the idea that $cday$ predicts better future stock returns than cay. In addition, our model beats the performance of the model that includes $cday$ only. In fact, at the four quarter horizon, $cday_t$ and $\Delta cday_t$ explain 26% (Belgium), 22% (France and UK), 17% (Canada), 13% (Denmark), 7% (Australia), 6% (Netherlands), 4% (Finland and US) of the real stock return, which is again lower than the adjusted R-square statistics associated with our model.

Table 7

Forecasting regressions for real stock returns (using *cay*): additional control variables.

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
	1	2	3	4		1	2	3	4
	Australia					Denmark			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.04]	[0.05]	[0.06]	[0.07]	$\bar{R}^2$	[0.07]	[0.11]	[0.14]	[0.15]
	Panel B: $cay + \Delta cay + r_{t-1} + dy$					Panel B: $cay + \Delta cay + r_{t-1}$			
$\bar{R}^2$	[0.10]	[0.13]	[0.16]	[0.19]	$\bar{R}^2$	[0.24]	[0.20]	[0.21]	[0.19]
	Austria					Finland			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.02]	[0.03]	[0.03]	[0.04]	$\bar{R}^2$	[0.04]	[0.07]	[0.11]	[0.14]
	Panel B: $cay + \Delta cay + r_{t-1}$					Panel B: $cay + \Delta cay + r_{t-1} + dy$			
$\bar{R}^2$	[0.03]	[0.04]	[0.04]	[0.04]	$\bar{R}^2$	[0.09]	[0.20]	[0.27]	[0.29]
	Belgium					France			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.11]	[0.11]	[0.12]	[0.08]	$\bar{R}^2$	[0.07]	[0.13]	[0.18]	[0.20]
	Panel B: $cay + r_{t-1} + dy$					Panel B: $cay + r_{t-1} + dy$			
$\bar{R}^2$	[0.19]	[0.26]	[0.31]	[0.32]	$\bar{R}^2$	[0.14]	[0.19]	[0.17]	[0.19]
	Canada					Germany			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.07]	[0.11]	[0.07]	[0.17]	$\bar{R}^2$	[0.01]	[0.01]	[0.02]	[0.02]
	Panel B: $cay + \Delta cay + r_{t-1} + dy$					Panel B: $cay + \Delta cay + r_{t-1} + dy$			
$\bar{R}^2$	[0.14]	[0.20]	[0.20]	[0.17]	$\bar{R}^2$	[0.05]	[0.10]	[0.13]	[0.12]

Table 8

Forecasting regressions for real stock returns (using cay): additional control variables (cont.).

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
	1	2	3	4		1	2	3	4
	Ireland					Spain			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.00]	[0.00]	[0.00]	[0.00]	$\bar{R}^2$	[0.00]	[0.00]	[0.00]	[0.00]
	Panel B: $cay + \Delta cay + r_{t-1}$					Panel B: $cay + \Delta cay + r_{t-1}$			
$\bar{R}^2$	[0.01]	[0.01]	[0.00]	[0.00]	$\bar{R}^2$	[0.01]	[0.02]	[0.04]	[0.02]
	Italy					Sweden			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.01]	[0.01]	[0.02]	[0.02]	$\bar{R}^2$	[0.00]	[0.00]	[0.00]	[0.00]
	Panel B: $cay + \Delta cay + r_{t-1} + dy$					Panel B: $cay + \Delta cay + r_{t-1} + dy$			
$\bar{R}^2$	[0.21]	[0.28]	[0.36]	[0.40]	$\bar{R}^2$	[0.21]	[0.31]	[0.38]	[0.41]
	Japan					UK			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.05]	[0.05]	[0.04]	[0.04]	$\bar{R}^2$	[0.09]	[0.15]	[0.15]	[0.18]
	Panel B: $cay + r_{t-1} + dy$					Panel B: $cay + \Delta cay + r_{t-1} + dy$			
$\bar{R}^2$	[0.06]	[0.08]	[0.08]	[0.09]	$\bar{R}^2$	[0.08]	[0.15]	[0.20]	[0.28]
	Netherlands					US			
	Panel A: cay only					Panel A: cay only			
$\bar{R}^2$	[0.02]	[0.02]	[0.04]	[0.05]	$\bar{R}^2$	[0.03]	[0.06]	[0.07]	[0.08]
	Panel B: $cay + \Delta cay + r_{t-1} + dy$					Panel B: $cay + \Delta cay + r_{t-1} + dy$			
$\bar{R}^2$	[0.10]	[0.20]	[0.27]	[0.32]	$\bar{R}^2$	[0.06]	[0.08]	[0.10]	[0.11]

Table 9

Forecasting regressions for real stock returns (using *cday*): additional control variables.

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
	1	2	3	4		1	2	3	4
	Australia					Denmark			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.03]	[0.04]	[0.05]	[0.07]	$\bar{R}^2$	[0.06]	[0.10]	[0.12]	[0.13]
	Panel B: $cday + r_{t-1} + dy$					Panel B: $cday + r_{t-1}$			
$\bar{R}^2$	[0.08]	[0.11]	[0.15]	[0.18]	$\bar{R}^2$	[0.23]	[0.18]	[0.19]	[0.17]
	Austria					Finland			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.01]	[0.01]	[0.02]	[0.02]	$\bar{R}^2$	[0.03]	[0.03]	[0.04]	[0.04]
	Panel B: $cday + r_{t-1}$					Panel B: $cday + r_{t-1} + dy$			
$\bar{R}^2$	[0.03]	[0.03]	[0.03]	[0.02]	$\bar{R}^2$	[0.06]	[0.21]	[0.17]	[0.13]
	Belgium					France			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.18]	[0.24]	[0.28]	[0.26]	$\bar{R}^2$	[0.07]	[0.14]	[0.19]	[0.22]
	Panel B: $cday + r_{t-1} + dy$					Panel B: $cday + r_{t-1} + dy$			
$\bar{R}^2$	[0.22]	[0.30]	[0.34]	[0.34]	$\bar{R}^2$	[0.12]	[0.17]	[0.24]	[0.30]
	Canada					Germany			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.07]	[0.19]	[0.19]	[0.17]	$\bar{R}^2$	[0.06]	[0.05]	[0.03]	[0.02]
	Panel B: $cday + r_{t-1} + dy$					Panel B: $cday + r_{t-1} + dy$			
$\bar{R}^2$	[0.17]	[0.25]	[0.28]	[0.25]	$\bar{R}^2$	[0.08]	[0.09]	[0.08]	[0.07]

Table 10

Forecasting regressions for real stock returns (using *cday*): additional control variables (cont.).

The dependent variable is H-period real return $r_{t+1} - r_{f,t+1} + ... + r_{t+H} - r_{f,t+H}$,

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

Newey-West (1987) corrected t-statistics appear in parenthesis.

	Forecast Horizon H					Forecast Horizon H			
	1	2	3	4		1	2	3	4
	Ireland					Spain			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.00]	[0.00]	[0.00]	[0.00]	$\bar{R}^2$	[0.00]	[0.00]	[0.01]	[0.03]
	Panel B: $cday + r_{t-1}$					Panel B: $cday + r_{t-1}$			
$\bar{R}^2$	[0.01]	[0.01]	[0.00]	[0.01]	$\bar{R}^2$	[0.02]	[0.04]	[0.10]	[0.09]
	Italy					Sweden			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.01]	[0.02]	[0.02]	[0.03]	$\bar{R}^2$	[0.03]	[0.08]	[0.11]	[0.11]
	Panel B: $cday + r_{t-1} + dy$					Panel B: $cday + r_{t-1} + dy$			
$\bar{R}^2$	[0.13]	[0.21]	[0.31]	[0.37]	$\bar{R}^2$	[0.18]	[0.24]	[0.28]	[0.30]
	Japan					UK			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.05]	[0.05]	[0.04]	[0.04]	$\bar{R}^2$	[0.06]	[0.12]	[0.17]	[0.22]
	Panel B: $cday + r_{t-1} + dy$					Panel B: $cday + r_{t-1} + dy$			
$\bar{R}^2$	[0.05]	[0.07]	[0.08]	[0.21]	$\bar{R}^2$	[0.13]	[0.17]	[0.21]	[0.26]
	Netherlands					US			
	Panel A: $cday$ only					Panel A: $cday$ only			
$\bar{R}^2$	[0.02]	[0.03]	[0.05]	[0.06]	$\bar{R}^2$	[0.01]	[0.02]	[0.03]	[0.04]
	Panel B: $cday + r_{t-1} + dy$					Panel B: $cday + r_{t-1} + dy$			
$\bar{R}^2$	[0.18]	[0.32]	[0.37]	[0.38]	$\bar{R}^2$	[0.02]	[0.03]	[0.02]	[0.04]

3.5 Nested forecast comparisons

As a final robustness exercise, we make nested forecast comparisons, in which we compare the mean-squared forecasting error from a series of one-quarter-ahead out-of-sample forecasts obtained from a prediction equation that includes either cay and Δcay or $cday$ and $\Delta cday$ as the only forecasting variables, to a variety of forecasting equations that do not include these variables.

We consider two benchmark models: the autoregressive benchmark and the constant expected returns benchmark. In the autoregressive benchmark, we compare the mean-squared forecasting error from a regression that includes just the lagged asset return as a predictive variable to the mean-squared error from regressions that include, in addition, cay and Δcay or $cday$ and $\Delta cday$. In the constant expected returns benchmark, we compare the mean-squared forecasting error from a regression that includes a constant to the mean-squared error from regressions that include, in addition, cay and Δcay or $cday$ and $\Delta cday$.

A summary of the nested forecast comparisons for the equations of the real stock returns using respectively cay and Δcay or $cday$ and $\Delta cday$ is provided in Tables 11 and 12. In general, including cay and Δcay in the forecasting regressions improves vis-a-vis the benchmark models. This is especially true in the case of the of the constant expected returns benchmark, supporting the evidence that reports time-variation in expected returns.

In addition, the models that include $cday$ and $\Delta cday$ generally have a lower mean-squared forecasting error. Moreover, the ratios are smaller that the ones presented in Table 11, reflecting the better predicting ability for stock returns of $cday$ and $\Delta cday$ relative to cay and Δcay.

Table 11

Forecasting regressions for real stock returns (using *cay*): nested forecast comparisons.

MSE represents the mean-squared forecasting error

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

	$\mathrm{MSE}_{\mathrm{cay}+\Delta\mathrm{cay}}/\mathrm{MSE}_{\mathrm{constant}}$	$\mathrm{MSE}_{\mathrm{cay}+\Delta\mathrm{cay}}/\mathrm{MSE}_{\mathrm{AR}}$
Australia	0.97	0.98
Austria	1.01	1.01
Belgium	0.96	0.94
Canada	0.99	0.99
Denmark	0.97	0.99
Finland	0.98	1.00
France	0.98	1.00
Germany	1.07	1.07
Ireland	1.02	1.02
Italy	1.01	1.01
Japan	0.88	0.88
Netherlands	1.00	1.00
Spain	0.93	0.95
Sweden	1.01	1.01
UK	1.00	1.00
US	0.99	0.99

Table 12

Forecasting regressions for real stock returns (using *cday*): nested forecast comparisons.

MSE represents the mean-squared forecasting error

Symbols ***, **, and * represent significance at a 1%, 5% and 10% level, respectively.

	$\mathrm{MSE}_{\mathrm{cday}+\Delta\mathrm{cday}}/\mathrm{MSE}_{\mathrm{constant}}$	$\mathrm{MSE}_{\mathrm{cday}+\Delta\mathrm{cday}}/\mathrm{MSE}_{\mathrm{AR}}$
Australia	0.98	0.99
Austria	1.02	1.02
Belgium	0.92	0.93
Canada	0.97	0.96
Denmark	0.98	1.00
Finland	1.01	1.00
France	0.97	1.00
Germany	1.04	1.05
Ireland	1.02	1.02
Italy	1.00	1.01
Japan	0.89	0.88
Netherlands	1.00	1.00
Spain	0.93	0.94
Sweden	0.97	0.98
UK	1.00	1.01
US	0.96	0.99

4 Conclusion

This paper uses the representative consumer's budget constraint, combines it with Epstein-Zin preferences and the homogeneity of the Bellman Equation and derives a relationship between expected excess returns, consumption growth, the consumption-aggregate wealth ratio, *cay*, and the first-order differences of this ratio, Δcay. We then explore this relationship to check whether it carries relevant information to predict future asset returns and explain the time-series of real stock returns.

When we use the consumption growth, *cay* and Δcay as conditioning variables for the Consumption-Capital Asset Pricing model (C-CAPM), we obtain a linear factor model that rivals the Lettau and Ludvigson (2001b) three-factor model in explaining expected returns. Moreover, the conditional factor model proposed is robust to the inclusion of additional control variables and in the context of nested forecasting comparisons.

Using data for 16 OECD countries, we show that the predictive ability of the model with regard to future real stock returns is stronger for Australia, Belgium, Canada, Denmark, Finland, the UK and the US. In the case of Germany, Ireland, and Spain, the evidence suggests that the model does not capture well the time-variation in stock returns.

The success of the model in predicting asset returns is due to its ability to track time-varying equilibrium risk premia. The model: (i) captures the fact that investors try to "smooth out" transitory movements in their asset wealth arising from time variation in expected returns; and (ii) shows that agents with low intertemporal elasticity of substitution and high risk aversion demand large equity risk premia because they fear that a reduction in economic prospects or a rise in economic uncertainty will lower asset prices. The risks for the long-run are, therefore, important determinants of the risk premium.

References

Abel, A. (1999), "Risk premia and term premia in general equilibrium", *Journal of Monetary Economics*, 43, 3-33.

Abel, A. (1990), “Asset prices under habit formation and catching up with the joneses”, *American Economic Review Papers and Proceedings*, 80(2), 38-42.

Ahn, S. C.; Gadarowski, C. (1999), “Small sample properties of the model specification test based on the Hansen-Jagannathan Distance”, Arizona State University, manuscript.

Aït-Sahalia, Y.; Parker, J. A.; Yogo, M. (2004), “Luxury goods and the equity premium”, *Journal of Finance*, 59, 2959–3004.

Attanasio, O.; Weber, G. (1989), "Intertemporal substitution, risk aversion and the Euler equation for consumption", *Economic Journal*, 99(395), 59-73.

Bansal, R.; Dittmar, R. F.; Lundblad, C. T. (2005), "Consumption, dividends, and the cross section of equity returns", *Journal of Finance*, 60(4), 1639-1672.

Bansal, R.; Gallant, A.; Tauchen, G. (2007), "Rational pessimism, rational exuberance, and asset pricing models", *Review of Economic Studies*, 74(4), 1005-1033.

Bansal, R.; Kiku, D.; Yaron, A. (2006), "Risks for the long-run: estimation and inference", Duke University, working paper.

Bansal, R.; Yaron, A. (2004), "Risks for the long run: a potential resolution of asset pricing puzzles", *Journal of Finance*, 59(4), 1481-1509.

Bekaert, G.; Engstrom, E.; Xing, S. (2005), "Risk, uncertainty and asset prices", Columbia University, working paper.

Benzoni, L.; Goldstein, R.; Collin-Dufresne, P. (2005), "Portfolio choice over the life cycle in the presence of 'trickle-down' labor income", University of Minnesota and University of California at Berkeley, working paper.

Bertaut, C. (2002), "Equity prices, household wealth, and consumption growth in foreign industrial countries: Wealth effects in the 1990s", Federal Reserve Board, International Finance Discussion Paper Working Paper No. 724.

Boudoukh, J.; Michaely, R.; Richardson, M.; Roberts, M. R. (2007). "On the importance of measuring payout yield: Implications for asset pricing", *Journal of Finance*, 62(2), 877-915.

Breeden, D. T. (1979), "An intertemporal asset pricing model with stochastic consumption and investment opportunities", *Journal of Financial Economics*, 7, 265-296.

Breeden, D. T.; Gibbons, M. R.; Litzenberger, R. H. (1989), "Empirical tests of the consumption-oriented CAPM", *Journal of Finance*, 44, 231–62.

Browning, M.; Hansen, L. P.; Heckman, J. J. (2000), "Micro data and general equilibrium models," in Taylor, J. B.; Woodford, M. (eds), *Handbook of Macroeconomics*, 1, Elsevier Science, North-Holland, Amsterdam.

Campbell, J. Y. (1996), "Understanding risk and return", *Journal of Political Economy*, 104, 298–345.

Campbell, J. Y. (1999), "Asset prices, consumption and the business cycle", in Taylor, J. B.; Woodford, M. (eds), *Handbook of Macroeconomics*, 1, Elsevier Science, North-Holland, Amsterdam.

Campbell, J.; Cochrane, J. (1999), "By force of habit: a consumption-based explanation of aggregate stock market behavior", *Journal of Political Economy*, 107(2), 205-251.

Campbell, J.; Shiller, R. (1988), "The dividend price ratio and expectation of future dividends and discount factors", *Review of Financial Studies*, 1(3), 195-227.

Catte, P.; Girouard, N.; Price, R.; Christophe, A. (2004). "Housing market, wealth and the business cycle", OECD Working Paper No. 17.

Chen, X.; Ludvigson, S. (2007), "Land of addicts? An empirical investigation of habit-based asset pricing models", New York University, working paper.

Cochrane, J. H. (1996), "A cross-sectional test of an investment-based asset pricing model", *Journal of Political Economy*, 104, 572–621.

Cochrane, J. H. (2001), *Asset pricing*, Princeton University Press, Princeton, New Jersey.

Constantinides, G. (1990), "Habit-formation: a resolution of the equity premium puzzle", *Journal of Political Economy*, 98(3), 519-543.

D'Avolio, G. (2002), "The market for borrowing stock", *Journal of Financial Economics*, 66(2-3), 271–306.

Duffee, G. (2005), "Time variation in the covariance between stock returns and consumption growth", *Journal of Finance*, 60(4), 1673-1712.

Epstein, L.; Zin, S. (1989), "Substitution, risk aversion and the temporal behaviour of consumption and asset returns I: a theoretical framework", *Econometrica*, 57(4), 937-969.

Epstein, L.; Zin, S. (1991), "Substitution, risk aversion and the temporal behaviour of consumption and asset returns II: an empirical analysis", *Journal of Political Economy*, 99(2), 1263-1286.

Fama, E. (1970), "Efficient capital markets: a review of theory and empirical work", *Journal of Finance*, 25(2), 383-417.

Fama, E. (1991), "Efficient capital markets: II", *Journal of Finance*, 46(5), 1575-1617.

Fama, E. (1998), "Market efficiency, long term returns and behavioral finance", *Journal of Financial Economics* 49(3), 283-306.

Fama, E.; French, K. (1988), "Dividend yields and expected stock returns", *Journal of Financial Economics*, 22(1), 3-25.

Fama, E.; French„ K. (1992), "The cross-section of expected stock returns", *Journal of Finance*, 47(2), 427-465.

Fama, E.; French„ K. (1993), "Common risk factors in the returns on stocks and bonds", *Journal of Financial Economics*, 33(1), 3-56.

Fama, E.; French„ K. (1996), "Multifactor explanations of asset pricing anomalies", *Journal of Financial Economics*, 51(1), 55-84.

Fama, E.; Macbeth, J. (1973), “Risk, return, and equilibrium: empirical tests”, *Journal of Political Economy*, 81(3), 607-636.

Farmer, J.; Lo, A. (1999), "Frontiers of science: evolution and efficient markets", *Proceedings of the National Academy of Sciences*, 96(18), 9991-9992.

Fernandez-Corugedo, E.; Price, S.; Blake, A. (2003), "The dynamics of consumers' expenditure: the UK consumption ECM redux", Bank of England Working Paper #204.

Ferson, W.; Constantinides, G. (1991), “Habit persistence and durability in aggregate consumption: empirical tests”, *Journal of Financial Economics*, 29(2), 199-240.

Ferson, W.; Foerster, S. (1994), “Finite sample properties of the generalized method of moments in tests of conditional asset pricing models”, *Journal of Financial Economics*, 36(1), 29-55.

Goyal, A.; Welch, I. (2003), "Predicting the equity premium with dividend ratios", *Management Science*, 49(5), 639-654.

Guvenen, F. (2001), "Mismeasurement of the elasticity of intertemporal substitution: the role of limited stock market participation", University of Rochester, manuscript.

Hansen, L. P.; Jagannathan, R. (1997), "Assessing specification errors in stochastic discount factor models”, *Journal of Finance*, 52(2), 557-590.

Hansen, L. P.; Singleton, K. J. (1982), “Generalized instrumental variables estimation of nonlinear rational expectations models”, *Econometrica*, 50, 1269-1286.

Hansen, L. P.; Heaton, J.; Li, N. (2006), "Consumption strikes back", University of Chicago, working paper.

Hansen, L. P.; Heaton, J.; Yaron, A. (1996), "Finite-sample properties of some alternative GMM estimators”, *Journal of Business and Economic Statistics*, 14(3), 262-280.

Hall, R. E. (1988), “Intertemporal substitution in consumption”, *Journal of Political Economy*, 96(2), 339-357.

Jagannathan, R.; Wang, Z. (1996), "The conditional CAPM and the cross-section of expected returns", *Journal of Finance*, 51(1), 3-54.

Julliard, C. (2004), "Labor income risk and asset returns", Princeton University, manuscript.

Lamont, O. (1998), "Earnings and expected returns", *Journal of Finance*, 53, 1563-1587.

Lamont, O. A.; Thaler, R. H. (2003), "Can the market add and subtract? Mispricing in tech stock carve-outs", *Journal of Political Economy*, 111(2), 227-268.

Lettau, M.; Ludvigson, S. (2001a), "Consumption, aggregate wealth, and expected stock returns", *Journal of Finance*, 56(3), 815-849.

Lettau, M.; Ludvigson, S. (2001b), "Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying", *Journal of Political Economy*, 109, 1238-1286.

Lettau, M.; Ludvigson, S. (2004), "Understanding trend and cycle in asset values: reevaluating the wealth effect on consumption", *American Economic Review*, 94(1), 276-299.

Lintner, J. (1965), "The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets", *Review of Economics and Statistics*, 47, 13–37.

Lucas, R. E. (1978), "Asset prices in an exchange economy", *Econometrica*, 46, 1429-1445.

Lustig, H.; Van Nieuwerburgh, S. (2005), "Housing collateral, consumption insurance, and risk premia: an empirical perspective", *Journal of Finance*, 60(3), 1167-1219.

Malloy, C.; Moskowitz, T.; Vissing-Jorgensen (2005), "Job risk and asset returns", University of Chicago, working paper.

Mankiw, N. G.; Shapiro, M. D. (1986), "Risk and return: consumption beta versus market beta", *Review of Economics and Statistics*, 68, 452-59.

Mehra, R.; Prescott, E. (1985), "The equity premium: a puzzle", *Journal of Monetary Economics*, 15(2), 145-161.

Menzly, L.; Santos, T.; Veronesi, P. (2004), "Understanding predictability", *Journal of Political Economy*, 112(1), 1-47.

Merton, R. C. (1973), "An intertemporal capital asset pricing model", *Econometrica*, 41(5), 867-887.

Newey, W.; West, K. (1987), "A simple positive semi-definite, heterokedasticity, and autocorrelation consistent covariance matrix", *Econometrica*, 55(3), 703-708.

Pakos, M. (2003), "Asset pricing with durable goods and non-homothetic preferences", University of Chicago, manuscript.

Parker, J. A.; Julliard, C. (2005), "Consumption risk and the cross section of expected returns", *Journal of Political Economy*, 113(1), 185-222.

Piazzesi, M.; Schneider, M. (2006), "Equilibrium yield curves", NBER Working Paper #12609.

Piazzesi, M.; Schneider, M.; Tuzel, S. (2007), "Housing, consumption, and asset pricing", *Journal of Financial Economics*, 83(3), 531-569.

Pichette, L.; Tremblay, D. (2003). "Are wealth effects important for Canada?", Bank of Canada Working Paper No. 30.

Poterba, J.; Samwick, A. (1995). "Stock ownership patterns, stock market fluctuations, and consumption", *Brookings Papers on Economic Activity*, 2, 295-372.

Poterba, J.; Summers, L. (1988), "Mean reversion in stock prices: evidence and implications", *Journal of Financial Economics*, 22(1), 27-60.

Richards, A. (1995), "Comovements in national stock market returns: evidence of predictability, but not cointegration", *Journal of Monetary Economics*, 36(3), 631-654.

Robertson, D.; Wright, S. (2006). 'Dividends, total cash flow to shareholders and predictive return regressions", *Review of Economics and Statistics*, 88(1), 91-99.

Roll, R. (1977): "A critique of the asset pricing theory's test: Part 1: on past and potential testability of the theory", *Journal of Financial Economics*, 4, 129-176.

Rubinstein, M. (1976), "The valuation of uncertain income streams and the pricing of options", Bell Journal of Economics, 7(2), 407-425.

Santos, T.; Veronesi, P. (2006), "Labor income and predictable stock returns", *Review of Financial Studies*, 19(1), 1-44.

Sharpe, W. (1964), "Capital asset prices: a theory of market equilibrium under conditions of risk", *Journal of Finance*, 19, 425–442.

Sousa, R. M. (2010), "Consumption, (dis)aggregate wealth, and asset returns", *Journal of Empirical Finance*, 17(4), 606-622.

Sousa, R. M. (2012), "What is the impact of wealth shocks on asset allocation?", *Quantitative Finance*, forthcoming.

Sundaresan, S. M. (1989), "Intertemporally dependent preferences and the volatility of consumption and wealth", *Review of Financial Studies*, 2(1), 73-89.

Tan, A.; Voss, G. (2003). "Consumption and wealth in Australia", The Economic Record, 79(244), 39-56.

Vissing-Jorgensen (2002), "Limited asset market participation and the elasticity of intertemporal substitution", *Journal of Political Economy*, 110, 825-853.

Yang, W. (2007), "Time-varying exposure to long-run consumption risk", University of Rochester, working paper.

Yogo, M. (2006), "A consumption-based explanation of expected stock returns", *Journal of Finance*, 61(2), 539-580.

Wei, M. (2005), "Human capital, business cycles and asset pricing", Board of Governors of the Federal Reserve - Division of Monetary Affairs, working paper.

Appendix

A Combining Epstein-Zin Preferences with the Intertemporal Budget Constraint

With Epstein-Zin preferences, the utility function is defined recursively as

$$U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{14}$$

where C_t is the consumption, δ is the rate of time preference, γ is the relative risk aversion, $\theta := \frac{1-\gamma}{1-1/\psi}$, ψ is the intertemporal elasticity of substitution,and E_t is the rational expectation operator.

The budget constraint is

$$W_{t+1} = R_{t+1} \left(W_t - C_t \right) \quad \forall t$$

where W is total wealth and R_t is the return on wealth, that is,

$$R_{t+1} := \left(1 - \sum_{i=1}^{N} w_{it} \right) R^f + \sum_{i=1}^{N} w_{it} R_{it+1} = R^f + \sum_{i=1}^{N} w_{it} \left(R_{it+1} - R^f \right) \tag{15}$$

where w_i is the wealth share invested in the i^{th} risky asset and R^f is the risk-free rate.

The recursive structure of the utility function makes it straightforward to write down the Bellman equation, despite its non-linearity. The optimal value of the utility, V, at time t will be a function of the wealth W_t. From equation (12), we have that the Bellman equation takes the form

$$V\left(W_t\right) \equiv \max_{\{C,w\}} \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V\left(W_{t+1}\right)^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}.$$

By homogeneity,

$$V\left(W_t\right) \equiv \phi_t W_t$$

for some ϕ_t. Therefore, the first-order condition C_t will be

$$\begin{aligned}
(1-\delta) C_t^{\frac{1-\gamma}{\theta}-1} &= \delta \left(E_t \left[V\left(W_{t+1}\right)^{1-\gamma} \right] \right)^{\frac{1}{\theta}-1} E_t \left[V\left(W_{t+1}\right)^{-\gamma} \phi_{t+1} R_{t+1} \right] \\
&= \delta \left(E_t \left[\phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}-1} E_t \left[\phi_{t+1}^{1-\gamma} W_{t+1}^{-\gamma} R_{t+1} \right] \\
&= \delta E_t \left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} \left(W_t - C_t \right)^{\frac{1-\gamma}{\theta}-1}.
\end{aligned} \tag{16}$$

where we simplified terms before writing the first line and used the budget constraint to substitute out W_{t+1} in the last line.

Given the structure of the problem, consumption is also proportional to W_t, that is $C_t = \omega_t W_t$. Therefore the last equation can be rewritten as

$$(1-\delta)\,\omega_t^{\frac{1-\gamma}{\theta}-1} = \delta E_t\left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}} (1-\omega_t)^{\frac{1-\gamma}{\theta}-1}$$

$$\rightarrow \delta E_t\left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}} = (1-\delta)\left(\frac{\omega_t}{1-\omega_t}\right)^{\frac{1-\gamma}{\theta}-1} \tag{17}$$

We can now rewrite the Bellman equation using homogeneity and the last result as

$$\begin{aligned}
\phi_t &= \max\left\{(1-\delta)\left(\frac{C_t}{W_t}\right)^{\frac{1-\gamma}{\theta}} + \delta\left(E_t\left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\left(1-\frac{C_t}{W_t}\right)^{\frac{1-\gamma}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \\
&= \max\left\{(1-\delta)\,\omega_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t\left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}(1-\omega_t)^{\frac{1-\gamma}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \\
&= (1-\delta)^{\frac{\theta}{1-\gamma}}\left\{\omega_t^{\frac{1-\gamma}{\theta}} + \left(\frac{\omega_t}{1-\omega_t}\right)^{\frac{1-\gamma}{\theta}-1}(1-\omega_t)^{\frac{1-\gamma}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \\
&= (1-\delta)^{\frac{\theta}{1-\gamma}}\,\omega_t^{1-\frac{\theta}{1-\gamma}} = (1-\delta)^{\frac{\theta}{1-\gamma}}\left(\frac{C_t}{W_t}\right)^{1-\frac{\theta}{1-\gamma}}
\end{aligned}$$

where the budget constraint is used to replace W_{t+1} in the first line, and in the third line the max operator is removed since $\delta E_t\left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}$ is replaced with its value coming from the first-order condition (15). Plugging the solution for ϕ_t in the first-order condition (14) we can derive the Euler equation for the return on wealth

$$\begin{aligned}
1 &= \frac{\delta}{1-\delta}E_t\left[\phi_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\left(\frac{W_t}{C_t}-1\right)^{\frac{1-\gamma}{\theta}-1} = \delta E_t\left[\left(\frac{C_{t+1}}{W_{t+1}}\right)^{1-\gamma-\theta} R_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\left(\frac{W_t}{C_t}-1\right)^{\frac{1-\gamma}{\theta}-1} \\
&= \delta E_t\left[\left(\frac{C_{t+1}}{W_{t+1}}\right)^{1-\gamma-\theta}\left(\frac{W_t}{C_t}-1\right)^{1-\gamma-\theta} R_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}} \\
&= \delta E_t\left\{\left[\frac{C_{t+1}}{C_t}\left(\frac{W_t-C_t}{W_{t+1}}\right)\right]^{1-\gamma-\theta} R_{t+1}^{1-\gamma}\right\}^{\frac{1}{\theta}} = \delta E_t\left\{\left[\frac{C_{t+1}}{C_t}R_{t+1}^{-1}\right]^{1-\gamma-\theta} R_{t+1}^{1-\gamma}\right\}^{\frac{1}{\theta}}
\end{aligned}$$

$$\rightarrow 1 = E_t\left[\delta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{t+1}^{\theta}\right] \quad \forall t \tag{18}$$

The first-order condition for w_{it} is

$$\begin{aligned}
E_t\left[\phi_{t+1}^{1-\gamma}R_{t+1}^{-\gamma}\left(W_t-C_t\right)^{-\gamma}\left(R_{it+1}-R^f\right)\right] &= 0\\
E_t\left[\left(\frac{C_{t+1}}{W_{t+1}}\right)^{1-\gamma-\theta}R_{t+1}^{-\gamma}\left(R_{it+1}-R^f\right)\right] &= 0\\
E_t\left[\left(\frac{C_{t+1}}{W_{t+1}}\right)^{1-\gamma-\theta}\left(\frac{W_t}{C_t}-1\right)^{1-\gamma-\theta}R_{t+1}^{-\gamma}\left(R_{it+1}-R^f\right)\right] &= 0\\
E_t\left\{\left[\frac{C_{t+1}}{C_t}R_{t+1}^{-1}\right]^{1-\gamma-\theta}R_{t+1}^{-\gamma}\left(R_{it+1}-R^f\right)\right\} &= 0\\
E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}\left(R_{it+1}-R^f\right)\right] &= 0
\end{aligned}$$

$$\therefore E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}R_{it+1}\right]=E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}\right]R^f \quad \forall t,i \tag{19}$$

where in the fourth line the budget constraint is used to substitute out W_{t+1}. From the Euler equation (16) and the definition of return on wealth (13) we have

$$1=E_t\left[\delta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}\left(R^f+\sum_{i=1}^{N}w_{it}\left(R_{it+1}-R^f\right)\right)\right] \quad \forall t$$

and using (17) to substitute out $E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}R_{it+1}\right\}$ and simplifying we have that the equilibrium risk free rate is such that:

$$1/R^f=E_t\left[\delta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}\right] \quad \forall t.$$

Multiplying both sides of (17) by δ^θ and using the last result to remove R^f, we have the Euler equation for any risky asset i :

$$E_t\left[\delta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}R_{t+1}^{\theta-1}R_{it+1}\right]=1 \quad \forall t,i.$$

B From the Intertemporal Budget Constraint to the Stochastic Discount Factor

Following Campbell (1996) and Jagannathan and Wang (1996), labor income (Y_t) can be thought of as the dividend on human capital (H_t). Under this assumption, the return to human capital can be defined as:

$$1+R_{h,t+1}=\frac{H_{t+1}+Y_{t+1}}{H_t}. \tag{20}$$

Under the assumption that the steady state human capital-labor income ratio is constant ($Y/H = \rho_h^{-1} - 1$, where $0 < \rho_h < 1$), this relation can be log-linearized around the steady state to get

$$r_{h,t+1} = (1-\rho_h)k_h + \rho_h(h_{t+1} - y_{t+1}) - (h_t - y_t) + \Delta y_{t+1} \tag{21}$$

where $r := log(1+R)$, $h := logH$, $y := logY$, k_h is a constant of no interest, and the variables without time subscript are evaluated at their steady state value. Assuming that $\lim_{i\to\infty} \rho_h^i(h_{t+i} - y_{t+i}) = 0$, the log human capital income ratio can be rewritten as a linear combination of future labor income growth and future returns on human capital:

$$h_t - y_t = \sum_{i=1}^{\infty} \rho_h^{i-1}(\Delta y_{t+i} - r_{h,t+i}) + k_h. \tag{22}$$

Equation (20) tells us that the log human capital to labor income ration ratio has to be equal to the discounted sum of future labor income growth and human capital returns. Moreover, this equation is similar, both in structure and interpretation, to the relation between the log dividend-price ratio and future returns and dividends derived by Campbell and Shiller (1988): taking time t conditional expectation of both sides, when the log human capital to labor income ratio is high, agents should expect high future labor income growth or low human capital returns.[6]

Defining W_t as aggregate wealth (given by human capital plus asset holdings), C_t as consumption, and $R_{w,t+1}$ as the return on aggregate wealth between period t and $t+1$, the consumer's budget constraint can be written as:[7]

$$W_{t+1} = (1 + R_{w,t+1})(W_t - C_t). \tag{23}$$

Campbell and Mankiw (1989) show that, under the assumption that the consumption-aggregate wealth is stationary and that $\lim_{i\to\infty} \rho_w^i(c_{t+i} - w_{t+i}) = 0$, where $\rho_w := (W - C)/W < 1$, equation (21) can be approximated by Taylor expansion obtaining

$$c_t - w_t = \sum_{i=1}^{\infty} \rho_w^i r_{w,t+i} - \sum_{i=1}^{\infty} \rho_w^i \Delta c_{t+i} + k_w \tag{24}$$

where $c := logC$, $w := logW$, and k_w is a constant. The aggregate return on wealth can be decomposed as

$$R_{w,t+1} = \omega_t R_{a,t+1} + (1 - \omega_t)R_{h,t+1} \tag{25}$$

[6] Campbell and Shiller (1988), defining the log return of an asset as $r_t = log(P_t + D_t) - logP_{t-1}$, (where P and D are, respectively, price and dividend of the asset) derive the relation $d_t - p_t = E_t \sum_{i=1} \rho^{i-1}(r_{t+i} - \Delta d_{t+i}) + k_d$ where $d := \log d$ and $p := \log P$.

[7] Labor income does not appear explicitly in this equation because of the assumption that the market value of tradable human capital is included in aggregate wealth.

where ω_t is a time varying coefficient and $R_{a,t+1}$ is the return on asset wealth. Campbell (1996) shows that we can approximate this last expression as

$$r_{w,t} = \omega r_{a,t} + (1-\omega) r_{h,t} + k_r \tag{26}$$

where k_r is a constant, ω is the mean of ω_t and $r_{w,t}$ is the log return on asset wealth. Moreover, we can approximate the log total wealth as

$$w_t = \omega a_t + (1-\omega) h_t + k_a \tag{27}$$

where a_t is the log asset wealth and k_a is a constant.

Replacing equation (20), (24), and (25) into (21), we get

$$c_t - \omega a_t - (1-\omega)(y_t + \sum_{i=1}^{\infty} \rho_h^{i-1} \Delta y_{t+i}) =$$

$$= \sum_{i=1}^{\infty} \rho_w^i (\omega r_{a,t+i} - \Delta c_{t+i}) + (1-\omega) \sum_{i=1}^{\infty} (\rho_w^i - \rho_h^{i-1}) r_{h,t+i} + k. \tag{28}$$

where k is a constant. This equation holds ex-post as a direct consequence of agent's budget constraint, but it also has to hold ex-ante. Taking time t conditional expectation of both sides, we have that

$$\underbrace{c_t - \omega a_t - (1-\omega) y_t}_{cay_t} - (1-\omega) \underbrace{E_t \sum_{i=1}^{\infty} \rho_h^{i-1} \Delta y_{t+i}}_{lr_t} = E_t \sum_{i=1}^{\infty} \rho_w^i (\omega r_{a,t+i} - \Delta c_{t+i}) + \eta_t + k$$

where: $lr_t := E_t \sum_{i=1}^{\infty} \rho_h^{i-1} \Delta y_{t+i}$ represent the expected growth in future labor income, this is, the labor income risk;[8] $\eta_t := (1-\omega) \sum^{\infty} (\rho_w^i - \rho_h^{i-1}) r_{h,t+i}$ is a stationary component; and, following Lettau and Ludvigson (2001a, 2001b), $cay_t := c_t - \omega a_t - (1-\omega) y_t$.

From the intertemporal budget constraint

$$\begin{aligned} R_{t+1}^{-1} &= \frac{W_t}{W_{t+1}} - \frac{C_t}{W_{t+1}} = \frac{C_t}{C_{t+1}} \left(\frac{W_t}{C_t} \frac{C_{t+1}}{W_{t+1}} - \frac{C_{t+1}}{W_{t+1}} \right) \\ \therefore \quad & R_{t+1}^{\theta-1} = e^{(\theta-1)\Delta c_{t+1}} \left[e^{\Delta c w_{t+1}} - e^{c w_{t+1}} \right]^{1-\theta} \end{aligned}$$

where $cw_t := \log(C_t / W_t)$.

[8] Following Campbell and Shiller (1988) and approximating the log return on human capital as $r_{h,t+1} = r + (E_{t+1} - E_t) \sum_{i=1}^{\infty} \rho_h^{i-1} \Delta y_{t+i}$, we have from equation (18) that the log human capital will depend only (disregarding constant terms) on current and future expected labor income

$$h_t = y_t + E_t \sum_{i=1}^{\infty} \rho_h^{i-1} \Delta y_{t+i},$$

therefore the human capital wealth level will vary as expectations of future labor income change.

Putting the last result into the euler equation we have

$$E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left[e^{\Delta cw_{t+1}} - e^{cw_{t+1}} \right]^{1-\theta} \left(R_{it+1} - R^f \right) \right\} = 0$$

where the stochastic discount factor is

$$M_{t+1} \propto \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left[e^{\Delta cw_{t+1}} - e^{cw_{t+1}} \right]^{1-\frac{1-\gamma}{1-1/\psi}}$$

or to estimate this we need a proxy for cw .If we follow Lettau and Ludvigson we have

$$cw_t \approx \kappa + cay_t.$$

Alternatively, we have the complete expression (with $Y/H =: \rho_h^{-1} - 1$, where $0 < \rho_h < 1$):

$$cw_t \approx \kappa + cay_t - (1-\omega) E_t \sum_{i=1}^{\infty} \rho_h^{i-1} (\Delta y_{t+i} - r_{h,t+i}).$$

If we use the return on the market to proxy for the return on total wealth (as Epstein and Zin (1989, 1991) originally suggested) we have:

$$\begin{aligned} M_{t+1} &= \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{M,t+1}^{\theta-1} = \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1-\gamma}{\psi-1}} R_{M,t+1}^{\frac{1-\gamma}{1-1/\psi}-1} \\ &= \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1-\gamma}{\psi-1}} R_{M,t+1}^{\frac{-\gamma+1/\psi}{1-1/\psi}}. \end{aligned}$$